Soru: $4x^2-4xy+7y^2+12x+6y-9=0$

$x=x'+h$
$y=y'+k$ ötelemesini uygulayıp x ve y li terimleri yok edelim

$4AC-B^2=4.4.7-(-4)^2\neq 0$ olup teorem kullanılabilir

$\Phi_x(x,y)=8x-4y+12$ $\qquad$ $\Phi_x(h,k)=8h-4k+12=0$

$\Phi_y(x,y)=-4x+14y+6$ $\qquad$ $\Phi_y(h,k)=-4h+14k+6=0$

$$8h-4k=-12$$
$$2/\ -4h+14k=-6$$
$$\overline{\qquad 24k=-24 \qquad}\quad k=-1,\ h=-2$$

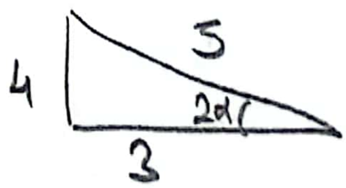
$F'=\Phi(-2,-1)=-24$

O halde denklem

$$4x'^2-4x'y'+7y'^2-24=0 \quad \text{bulunur.}$$

Şimdi $x'y'$ ifadesini yok etmek için dönme uygulayalım.

$$\tan 2\alpha=\frac{B}{A-C}=\frac{-4}{-3}=\frac{4}{3}$$



$\cos 2\alpha=\frac{3}{5}$

$\sin 2\alpha=\frac{4}{5}$

$$\cos 2\alpha=2\cos^2\alpha-1 \Rightarrow \cos^2\alpha=\frac{\frac{3}{5}+1}{2}=\frac{\cancel{8}^{4}}{\cancel{10}_{5}} \Rightarrow \cos\alpha=\pm\frac{2}{\sqrt5}$$

$$\sin 2\alpha=2\sin\alpha.\cos\alpha$$
$$\frac{4}{5}=2.\sin\alpha.\frac{2}{\sqrt5} \Rightarrow \sin\alpha=\frac{1}{\sqrt5}$$

$x' = x''\cos\alpha - y''\sin\alpha$

$y' = x''\sin\alpha + y''\cos\alpha$

$$x' = \frac{1}{\sqrt{5}}(2x'' - y'')$$

$$y' = \frac{1}{\sqrt{5}}(x'' + 2y'')$$

$$4 \cdot \frac{1}{5}(2x'' - y'')^2 - 4 \cdot \frac{1}{5}(2x'' - y'')(x'' + 2y'') + \frac{7}{5}(x'' + 2y'')^2 - 24 = 0$$

$$\frac{4}{5}(4x''^2 - 4x''y'' + y''^2) - \frac{4}{5}(2x''^2 + 4x''y'' - y''x'' - 2y''^2)$$

$$+ \frac{7}{5}(x''^2 + 4x''y'' + 4y''^2) - 24 = 0$$

$$\frac{16x''^2}{5} - \cancel{\frac{16x''y''}{5}} + \frac{4y''^2}{5} - \frac{8}{5}x''^2 - \cancel{\frac{16}{5}x''y''} + \cancel{\frac{4}{5}x''y''} + \frac{8y''^2}{5} + \frac{7}{5}x''^2$$

$$+ \cancel{\frac{28}{5}x''y''} + \frac{28}{5}y''^2 - 24 = 0$$

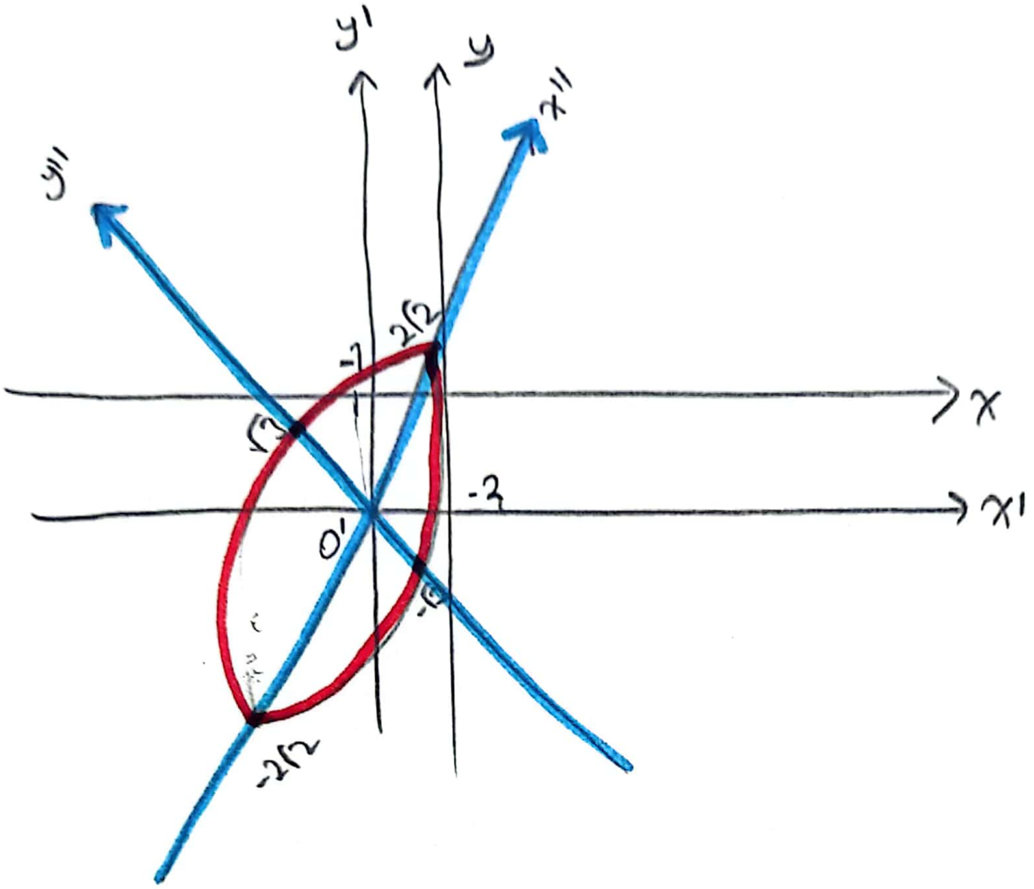
$$\frac{15x''^2}{5} + \frac{40}{5}y''^2 - 24 = 0$$

$$3x''^2 + 8y''^2 - 24 = 0 \Rightarrow \frac{3x''^2}{24} + \frac{8y''^2}{24} = 1$$

$$\frac{x''^2}{\frac{24}{3}} + \frac{y''^2}{\frac{24}{8}} = 1 \Rightarrow \frac{x''^2}{8} + \frac{y''^2}{3} = 1$$

$$\frac{x''^2}{(2\sqrt{2})^2} + \frac{y''^2}{(\sqrt{3})^2} = 1 \quad \text{Elips}$$

$\longrightarrow x1$


y'
y
x''
y''
2√2
-1
x
√2
-3
x'
O'
-√2
-2√2